PRIMORDIAL BLACK HOLES IN AN ACCELERATING UNIVERSE

David Polarski

Laboratoire de Physique Mathématique et Théorique, UMR 5825 CNRS,
Université de Montpellier II, 34095 Montpellier, France.

Laboratoire de Mathématiques et Physique Théorique, UMR 6083 CNRS,
Université de Tours, Parc de Grandmont, 37200 Tours, France.

Abstract

General expressions are given for the generation of Primordial Black Holes (PBH) in a universe with a presently accelerated expansion due to a(n effective) cosmological constant. We give expressions both for a powerlaw scalefree primordial spectrum and for spectra which are not of that type. Specializing to the case of a pure cosmological constant Λ and assuming flatness, we show that a comological constant with $\Omega_{\Lambda,0} = 0.7$ will decrease the mass variance at the PBH formation time by about 15% compared with a critical density universe.

PACS Numbers: 04.62.+v, 98.80.Cq

1 Introduction

The generation of a spectrum of primordial fluctuations in the very Early Universe is the crucial ingredient of all inflationary scenarios. These fluctuations can explain the generation of all (classical) inhomogeneities that can be seen in our universe, from the Cosmic Microwave Background (CMB) anisotropies to the Large Scale Structures (LSS) in the form of galaxies and clusters of galaxies. The inflationary paradigm therefore reconciles Big Bang cosmology with the appearance of an inhomogeneous universe [1]. In addition, each inflationary scenario makes accurate predictions allowing for observations to discriminate between the various model candidates. As the observations are ever increasing in variety and quality, it is now important for any theoretical framework aiming at an adequate description of the early universe to come up with accurate predictions of different kinds. One such prediction is the possible formation of Primordial Black Holes (PBH). Indeed, it was realized already some time ago that a spectrum of primordial fluctuations would lead to the production of PBH [2]. For this generation mechanism to be efficient, one typically needs a "blue" spectrum [3]. In this way, one can hope that the density contrast averaged over the Hubble radius is sufficiently large that the resulting PBH production is not unsignificant and can be used as a powerful constraint on the underlying spectrum of primordial fluctuations and therefore also on the inflationary model which generates them [4, 5]. The production of PBH takes place on scales much smaller than those probed by the CMB anisotropy and LSS formation. In this sense, it is analogous if less spectacular, to the generation of a primordial gravitational wave background in inflationary models. Of course in the latter case, its discovery would be a remarkable prediction of inflation while the existence of PBH is a confirmation of the existence of the primordial fluctuations spectrum spectrum itself, irrespective of the way it was generated, though we are aware that latest CMB observations strongly favour inflationary models with adiabatic primordial fluctuations. In a recent paper [6], it was shown that in earlier papers, the mass variance at very early times, when the PBH are assumed to have formed, was significantly overestimated. This must be corrected if one is to make accurate predictions in order to constrain the underlying high-energy particle physics inspired models. In our previous work we only considered a critical density universe. Here we want to generalize these results to a flat universe with non vanishing cosmological constant Λ. Some of our results apply to models with an effective cosmological constant as well. Indeed, recent Supernovae observations strongly suggest that we live in a presently accelerating universe with $\Omega_{m,0} \approx 0.3$, $\Omega_{\Lambda,0} \approx 0.7$, the inclusion of a(n effective) cosmological constant seems further to make all observations converge into a consistent picture. This is why it is important to extend our results about PBH formation in the presence of a cosmological constant. We also give a generalization to primordial perturbations spectra which are not scalefree, an interesting possibility to consider in view of the wide range of scales probed by PBH formation. We first review the formalism describing PBH formation.

2 PBH formation

We assume for simplicity that a PBH is formed when the density contrast averaged over a volume of the (linear) size of the Hubble radius satisfies $\delta_{min} \leq \delta \leq \delta_{max}$, and further that the PBH mass, M_{PBH}, is of the order of the "horizon mass" M_H, the mass contained inside the Hubble volume. Relyimg on semianalytic considerations it is common to take $\delta_{min} = \frac{1}{3}$, $\delta_{max} = 1$ but recent numerical simulations suggest rather $\delta_{min} \approx 0.7$ [7] and show that M_{PBH} can span a certain range, around M_H though, at a given formation time. More accurately, when some scale defined by the wavenumber k reenters the Hubble radius after inflation at some time t_k with $k = (aH)|_{t_k}$, it can lead to the production of PBH with $M_{PBH} \approx M_H(t_k)$. Obviously, there is a one-to-one correspondence between $\frac{a(t_k)}{k}$, $M_H(t_k)$, and k.

For Gaussian primordial fluctuations, the probability density $p_R(\delta)$, where δ is the density contrast averaged over a sphere of radius R, is given by

$$p_R(\delta) = \frac{1}{\sqrt{2\pi}\ \sigma(R)}\ e^{-\frac{\delta^2}{2\sigma^2(R)}}\ . \qquad (1)$$

Here, the dispersion (mass variance) $\sigma^2(R) \equiv \left\langle \left(\frac{\delta M}{M}\right)^2_R \right\rangle$ is computed using a top-hat window function,

$$\sigma^2(R) = \frac{1}{2\pi^2} \int_0^\infty \mathrm{d}k\ k^2\ W^2_{TH}(kR)\ P(k)\ , \qquad (2)$$

where $P(k)$ is the power spectrum (we assume isotropy of the ensemble). From a point of view of principles, the averages are quantum averages; however, an effective quantum-to-classical transition is achieved during inflation [8]. For PBHs produced by inflationary perturbations, this quantum-to-classical transition is guaranteed for all masses of interest to us (see [9]).

The expression $W_{TH}(kR)$ stands for the Fourier transform of the top-hat window function divided by the probed volume $V_W = \frac{4}{3}\pi R^3$,

$$W_{TH}(kR) = \frac{3}{(kR)^3}(\sin kR - kR\cos kR)\ . \qquad (3)$$

Hence the probability $\beta(M_H)$ that a region of comoving size $R = \frac{H^{-1}(t_k)}{a(t_k)}$ has an averaged density contrast at horizon crossing t_k in the range $\delta_{min} \leq \delta \leq \delta_{max}$, is given by

$$\beta(M_H) = \frac{1}{\sqrt{2\pi}\,\sigma_H(t_k)} \int_{\delta_{min}}^{\delta_{max}} e^{-\frac{\delta^2}{2\sigma_H^2(t_k)}}\,\mathrm{d}\delta \approx \frac{\sigma_H(t_k)}{\sqrt{2\pi}\,\delta_{min}} e^{-\frac{\delta_{min}^2}{2\sigma_H^2(t_k)}}\ , \qquad (4)$$

where $\sigma^2_H(t_k) \equiv \sigma^2(R)|_{t_k}$, and the last approximation is valid for $\delta_{min} \gg \sigma_H(t_k)$, and $(\delta_{max} - \delta_{min}) \gg \sigma_H(t_k)$.

Important conclusions can be drawn from (4). Let us consider first the value of $\beta(M_H)$ today. Today we have $\sigma_H^2(t_0) \simeq 10^{-8}$, so clearly the probability of forming a black hole today is extraordinarily small. This probability can increase in the primordial universe if the power is increased when we go backwards in time, but the probability will remain very small, $\beta(M_H) \ll 1$, at all times due to the magnitude of $\delta^2_{min}/\sigma^2_H(t_k) \gg 1$.

3 Mass variance in the presence of Λ

When the universe contains a cosmological constant Λ, this must be taken into account for a correct accurate calculation of the mass variance at early times. In this section, we will extend the formulas derived in [6]. As stressed there, one should distinguish the behaviour of the quantity $\sigma_H^2(t_k)$, which is ultimately the quantity of interest, from the quantity $k^3\phi^2(k,t_k)$ or $\delta_H^2(k,t_k)$ with

$$\delta_H^2(k,t_k) \equiv \frac{k^3}{2\pi^2}\ P(k,t_k) = \frac{2}{9\pi^2}\ k^3\ \Phi^2(k,t_k)\ , \qquad (5)$$

where t_k is the PBH formation time of interest, deep in the radiation dominated stage. However, when dealing with a flat universe with $\Omega_{m,0} < 1$, we have *today*

$$\delta_H^2(k_0,t_0) \equiv \frac{k_0^3}{2\pi^2}\ P(k_0,t_0) = \frac{2}{9\pi^2}\ \Omega_{m,0}^{-2}\ k_0^3\ \Phi^2(k_0,t_0)\ , \qquad (6)$$

where f_0 stands for any quantity evaluated today (at time t_0), $\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{cr,0}}$ is the present energy density of dust-like matter relative to the critical density and $\Omega_{\Lambda,0} \equiv \frac{\Lambda}{3H_0^2} = 1 - \Omega_{m,0}$. We first relate the quantities appearing in (5,6) at the formation time t_k and at the present time t_0 for arbitrary evolution of the universe after radiation domination and for a scalefree powerlaw spectrum.

General expressions with powerlaw spectrum: assuming a scalefree powerlaw primordial spectrum of the type $k^3\Phi^2(k) = A(t)\ k^{n-1}$ on super Hubble radius ("superhorizon") scales, we then have [10]

$$k^3\ \Phi^2(k,t_k) = \left(\frac{2}{3}\right)^2\ \left(1 - \frac{H}{a}\int_0^t a\ dt'\right)^{-2}_{t=t_0}\ k_0^3\ \Phi^2(k_0,t_0)\left(\frac{k}{k_0}\right)^{n-1}\ , \qquad (7)$$

and analogously

$$\delta_H^2(k,t_k) = \left(\frac{2}{3}\right)^2\ \left(1 - \frac{H}{a}\int_0^t a\, dt'\right)^{-2}_{t=t_0}\ \Omega_{m,0}^2\ \delta_H^2(k_0,t_0)\left(\frac{k}{k_0}\right)^{n-1}\ . \qquad (8)$$

The lower limit of integration in (7,8) can be safely taken to be zero. In (7,8), we have used a radiation dominated stage followed by some *arbitrary* evolution of the scale factor. In earlier work, we considered a radiation dominated stage followed by a matter dominated stage which constitutes a special case of (7,8). An effective cosmological constant as in quintessence models would also be a particular case of (7,8). However, in contrast to a pure cosmological constant Λ, the time evolution of the scale factor $a(t)$ is model-dependent and cannot be given in full generality. The quantity $k_0^3\ \Phi^2(k_0,t_0)$, or equivalently $\delta_H^2(k_0,t_0)$, at the present Hubble radius scale can be derived using the large angular scale CMB anisotropy data. It is that quantity that comes from observations which fixes the overall amplitude of the fluctuations spectrum. The COBE data show that it is of the following order of magnitude [11]

$$k_0^3\ \Phi^2(k_0,t_0) = 0.86\ \times 10^{-8}\ A_0^2(\{n_i\})\ , \tag{9}$$

where $A_0^2(\{n_i\})$ parametrizes the amplitude variations and is chosen such that

$$A_0^2(n=1,\Omega_{m,0}=0.3) \simeq 1 \qquad\qquad A_0^2(n=1,\Omega_{m,0}=1) \simeq 1.94\ . \tag{10}$$

The exact amplitude depends on the cosmological parameters $\{n_i\}$, referring to the background as well as to the inflationary perturbations, and this model dependence is encoded in the quantity of order unity $A_0(\{n_i\})$. Eq.(10) assumes a powerlaw spectrum with spectral index n at least on large scales. For fixed $n \neq 1$, while the absolute values in (10) are modified, the ratio between them is unaltered [11]. Note that for a quintessence model, $A_0^2(\{n_i\})$ is model dependent.

Finally, we must relate all our results to the quantity of interest for the computation of the PBH abundance, the mass variance $\sigma_H^2(t_k)$ on the Hubble radius scale at horizon crossing time t_k. As stressed in [6], one has (with $k = (aH)|_{t_k}$)

$$\sigma_H^2(t_k) \equiv \alpha^2(k)\ \delta_H^2(k,t_k)\ . \tag{11}$$

It is crucial to distinguish both quantities $\sigma_H(t_k)$ and $\delta_H(k,t_k)$. The quantity $\delta_H(k,t_k)$ can be reconstructed at the time t_k from its present value $\delta_H(k_0,t_0)$ using (8). But this is *not* the case for the quantity $\sigma_H(t_k)$ because the deformation of the power spectrum is different at the time t_k and today. In other words,

$$T(k',t_0) \neq T(k',t_k)\ , \tag{12}$$

where the transfer function $T(k,t)$ is defined through

$$P(k,t) = \frac{P(0,t)}{P(0,t_i)}\ P(k,t_i)\ T^2(k,t)\ , \quad T(k\to 0,t) \to 1\ . \tag{13}$$

Here, t_i is some initial time when all scales are outside the Hubble radius, $k \ll aH$, we can take for example $t_i = t_e$, the end of inflation. For a powerlaw scalefree spectrum we have [6]

$$\alpha^2(k) = \int_0^{\frac{k_e}{k}} x^{n+2}\ T^2(kx,t_k)\ W_{TH}^2(x)\ \mathrm{d}x\ , \tag{14}$$

where k_e corresponds to the shortest fluctuations wavelengths with the size of the Hubble radius at the end of inflation. The transfer function $T(k',t)$ in the integrand of (14) must be taken *at the time* t_k, not today. The accurate value of $\alpha(k)$ requires numerical calculations but estimates made in [6] show a significant overestimation of the mass variance $\sigma_H(t_k)$ when it is not computed correctly using the right quantity $\alpha(k)$. One then gets that $(10/9)^2\alpha^2(k) \ll 25$ (the value usually taken in the literature for a critical density universe) for all the mass range of PBH produced in the radiation era. If one is willing to use PBH formation as a precision tool in cosmology, it is important to check in how far the presence of a cosmological constant with $\Omega_{\Lambda,0} = 0.7$ brings further modifications.

An important conclusion can be immediately drawn by inspection of the integrand in (14) without accurate knowledge of the transfer function at time t_k. Indeed, as we are interested in times $t_k \ll t_{eq}$ and in universes where Ω_Λ domination occurs late, it is clear that at the time t_k, neither the long-wave nor the short-wave fluctuation modes are affected in any way by the presence of (an effective) Λ. As can be seen from (13), this implies that the transfer function at time t_k does not depend on Λ and the same must apply therefore to the quantity $\alpha(k)$. We conclude that the influence of a(n effective) cosmological constant on the probability $\beta(M_H)$ comes solely from its influence on the quantity $\delta^2_H(k,t_k)$, or $k^3\ \Phi^2(k,t_k)$. It is this influence that we will quantify in the next subsection. We now consider a powerlaw spectrum and specialize to a universe with a cosmological constant Λ.

Powerlaw spectrum with $\boldsymbol{\Lambda}$: in order to account for the presence of a cosmological constant Λ, we must replace the evolution of the scale factor $a(t)$ after the radiation dominated stage. The scale factor for this stage of the universe evolution is very well approximated by [12]

$$a(t) = a_1 \sinh^{2/3}(\beta t)\ , \tag{15}$$

where $\frac{2}{3}\beta = \sqrt{\Lambda/3} = H_0\sqrt{\Omega_{\Lambda,0}}$, a_1 =const. The evolution (15) smoothly interpolates between a pure (flat) dust-like matter dominated stage, with $a(t) \propto t^{\frac{2}{3}}$, for $\beta t \ll 1$ which is of course the case right after t_{eq}, and a Λ dominated universe in the asymptotic future. In particular for scales for which $z_{eq} > z(t_k) \gg 1$, $a(t_k) \propto t_k^{\frac{2}{3}}$. It is this evolution (15) which must be used in (7,8). It is physically appealing to express the results in terms of the quantity $M_H(t_k)$. Then the following result is obtained

$$\begin{aligned} k^3\ \Phi^2(k,t_k) \quad &= \quad \left(\frac{2}{3}\right)^2 \left(1 - \frac{H}{a}\int_0^t a\ dt'\right)^{-2}_{t=t_0} k_0^3\ \Phi^2(k_0,t_0) \\ &\times \quad \left[\frac{k_{eq}}{k_0}\right]^{n-1} \left[\frac{M_H(t_{eq})}{M_H(t_k)}\right]^{\frac{n-1}{2}}\ . \end{aligned} \tag{16}$$

The evolution (15) must now be substituted in (16). Actually it is slightly more accurate to write the quantity $\frac{k}{k_0}$ as

$$\frac{k}{k_0} \propto M_H(t_k)^{-\frac{1}{2}} \ , \qquad\qquad t_k \ll t_{eq} \tag{17}$$

where the proportionality constant is *independent* of $\Omega_{m,0}$ (and $\Omega_{\Lambda,0}$). We will take now the following quantities: $\Omega_{r,0} = 9.81 \times 10^{-5} \ \frac{g_{eff}}{3.36} \ h_{65}^{-2} \ (\frac{T_{\gamma,0}}{2.726})^4$ is the relative energy density of relativistic matter today, g_{eff} is the present effective number of relativistic degrees of freedom, $h_{65} \equiv \frac{H_0}{65}$km/s/Mpc, $T_{\gamma,0}$ is the present temperature of the CMB. Using these quantities, eq.(16) finally becomes for $\Omega_{\Lambda,0} = 0.7$

$$\begin{aligned} k^3 \ \Phi^2(k, t_k) \ &= \ 1.75 \times 10^{-8} \ \frac{0.219}{I^2(\Omega_{\Lambda,0})} \ A_0^2(\{n_i\}) \\ &\times \ \left[9.75 \times 10^{26} \ h_{65}^{-1} \ \left(\frac{g_{eff}}{3.36}\right)^{\frac{1}{4}} \frac{T_{\gamma,0}}{2.726^\circ K}\right]^{n-1} \left[\frac{\mathrm{g}}{M_H(t_k)}\right]^{\frac{n-1}{2}} \end{aligned} \tag{18}$$

where $M_H(t_k)$ is expressed in grams. In deriving 18 we have used

$$I(\Omega_{\Lambda,0}) \ \equiv \ 1 - \frac{H_0}{a_0} \int_0^{t_0} a(t) \ dt \simeq 1 - \frac{2}{3\sqrt{\Omega_{\Lambda,0}}} \int_0^d \left[\frac{\sinh x}{\sinh d}\right]^{\frac{2}{3}} dx \tag{19}$$

$$d \ \equiv \ \frac{1}{2} \ln \frac{1 + \sqrt{\Omega_{\Lambda,0}}}{1 - \sqrt{\Omega_{\Lambda,0}}} \ , \tag{20}$$

which gives the following numerical result substituted in (18)

$$I(0.7) = 0.468 \ . \tag{21}$$

Hence, $I^{-2}(0.7) = 4.57$ gives an increase of 64% compared to the value $25/9 = 2.78$ obtained for a critical density flat universe, $\Omega_{m,0} = 1$. On the other hand normalization to the CMB fluctuations decreases A_0^2 by a little bit more than 48%, as seen from (10). Therefore both effects combined lead to a decrease of about 15%. As for $\alpha(k)$, we have seen above that it is insensitive to the presence of a cosmological constant. From (17), the correspondence between the (approximate) PBH mass $M_H(t_k)$ and k is independent of $\Omega_{m,0}$ for $t_k \ll t_{eq}$. Therefore we conclude that, in a flat universe, a cosmological constant with $\Omega_{\Lambda,0} = 0.7$ will decrease the mass variance $\sigma_H^2(t_k)$ as follows

$$\sigma_H^2(t_k)|_{\Omega_{\Lambda,0}=0.7} \simeq 0.85 \ \sigma_H^2(t_k)|_{\Omega_{m,0}=1} \tag{22}$$

As a result, the significantly lower value estimated in [6] is further reduced by about 15%, further diminishing the probability for PBH formation. This is in good agreement with formulas presented in [13], where it is found that a nonzero cosmological constant relaxes the constraint on the spectral index n. It is interesting that these authors have incorrectly assumed that $\sigma_H(t_k) = C \ \delta_H(t_k)$ where the constant C is the same at all times t_k up to the present time t_0. Nevertheless, their eq.(4.30) would give the *relative* decrease of $\sigma_H(t_k)$ due to the presence of a cosmological constant. This is because the correct factor $\alpha(k)$ does not depend on Λ.

We finally note that equation (18) makes use of the observed amplitude today on the present Hubble radius scale and, as far as perturbations are concerned, combines it with an *assumed* (powerlaw) behaviour towards small scales on a very broad range of scales. Other behaviours are certainly possible however. This is reminiscent of the primordial gravitational wave background generated during inflation extending up to frequencies as high as 10^{10}Hz. There too, one can imagine a behaviour towards large frequencies departing from a simple scalefree law (see e.g. [14] for such a model with a jump in the tensorial spectral index n_T). Hence if the assumption of a scalefree spectrum does not hold, a more general expression will be needed. For this reason, we now generalize our results also to these cases.

Spectrum with a characteristic scale: A further important generalization concerns the primordial fluctuations spectrum itself. Indeed, the equations written in the previous subsection assume a scalefree spectrum. However, this needs not be the case especially in view of the large range of scales that are probed by PBH formation.

Let us therefore define in full generality

$$k^3 \ \Phi^2(k, t_k) \equiv \left(1 - \frac{H}{a}\int_0^t a \ dt'\right)^2_{t=t_k} \ F(k) = \frac{4}{9} \ F(k) \ , \tag{23}$$

where $F(k)$ can be any complicated function of k. The expression (23) represents the primordial spectrum on "super-Hubble radius" (superhorizon) scales. For example, the spectrum of double inflation considered in [10, 15] is of this general type. The corresponding generalization of (7), or (8), leads to a more complicated equation, viz.

$$\begin{aligned} k^3 \ \Phi^2(k, t_k) \ &= \ \left(\frac{2}{3}\right)^2 \ \left(1 - \frac{H}{a}\int_0^t a \ dt'\right)^{-2}_{t=t_0} \ k_0^3 \ \Phi^2(k_0, t_0) \\ &\times \ \frac{F(\alpha_1 M_H^{-\frac{1}{2}}(t_k))}{F(k_0)} \ , \end{aligned} \tag{24}$$

where α_1=constant. In case the function $F(k)$ is actually of the form $F(\frac{k}{k_s})$, where k_s defines the characteristic scale – an example of such a spectrum was found in [16] and considered in [17] – eq.(24) can be recast into a slightly simpler form

$$\begin{aligned} k^3 \ \Phi^2(k, t_k) \ &= \ \left(\frac{2}{3}\right)^2 \ \left(1 - \frac{H}{a}\int_0^t a \ dt'\right)^{-2}_{t=t_0} \ k_0^3 \ \Phi^2(k_0, t_0) \\ &\times \ \frac{F\left(\sqrt{\frac{M_s}{M_H(t_k)}}\right)}{F(k_0/k_s)} \end{aligned} \tag{25}$$

with $M_s \equiv M_H(t_{k_s})$ and $t_{k_s} < t_{eq}$. Specializing to the particular case $\Omega_{\Lambda,0} = 0.7$ just requires the substitution, like in (18), of the corresponding numbers into (24,25).

Finally, we come to the calculation of $\sigma_H(t_k)$ itself. We now have the corresponding generalization to primordial spectra of arbitrary shape

$$\sigma_H^2(t_k) = \frac{8}{81\pi^2} \int_0^{\frac{k_e}{k}} F(kx)\ x^3\ T^2(kx, t_k)\ W_{TH}^2(x)\ \mathrm{d}x\ . \tag{26}$$

In particular, the general expression for $\alpha^2(k)$ is given by

$$\alpha^2(k) = \int_0^{\frac{k_e}{k}} \frac{F(kx)}{F(k)}\ x^3\ T^2(kx, t_k)\ W_{TH}^2(x)\ \mathrm{d}x\ . \tag{27}$$

Again, for the case of interest to us, $t_k \ll t_{eq}$, the transfer function at the time t_k and therefore also $\alpha(k)$ are independent of Λ. We conclude that the same decrease found in (22) will apply here too. Note that (27) extends the result (14) derived in [6] for a powerlaw scalefree spectrum which just corresponds to $F(k) \propto k^{n-1}$. A characteristic scale in the primordial spectrum is an interesting possibility with respect to PBH formation in view of the large range of scales involved, much larger than CMB anisotropy or LSS formation. We have already considered some simple toy models in [6] and interesting results were obtained. The detailed numerical study of more sophisticated spectra and their possible relevance to observations is under progress [18].

References

[1] A. D. Linde, *Particle physics and inflationary cosmology* (Harwood, New York, 1990)
E. Kolb and M. Turner, *The Early Universe* (Addison-Wesley, Redwood City, 1990)
T. Padmanabhan, *Structure formation in the Universe* (Cambridge University Press, Cambridge, 1993)
A.R. Liddle and D.H. Lyth, *Cosmological inflation and large-scale structure* (Cambridge University Press, Cambridge, 2000)

[2] B. J. Carr, S. W. Hawking, Mon. Not. R. Astr. Soc. **168**, 399 1974
B. J. Carr, Astrophys. J. **205**, 1 1975
I. D. Novikov, A. G. Polnarev, A. A. Starobinsky, Ya. B. Zeldovich, Astronomy and Astrophysics **80**, 104 1979
B.J. Carr, in *Observational and theoretical aspects of relativistic astrophysics and cosmology*, edited by J.L. Sanz and L.J. Goicoechea (World Scientific, Singapore, 1985)

[3] B. J. Carr, J. E. Lidsey, Phys. Rev. D **48**, 543 1993
B.J. Carr, J.H. Gilbert, and J.E. Lidsey, Phys. Rev. D **50**, 4853 (1994)

[4] A. M. Green, A. R. Liddle, Phys. Rev. D **56**, 6166 1997; S. M. Leach, Ian J. Grivell, A. R. Liddle, Phys. Rev. D **62**, 043516 2000
Hee Il Kim, Phys. Rev. D **62**, 063504 2000
E. J. Copeland, A. R. Liddle, J. E. Lidsey, D. Wands, Phys. Rev. D **58**, 063508 1998
Hee Il Kim, Chul H. Lee, J. H. MacGibbon, Phys. Rev. D **59**, 063004 1999
J. Yokoyama, Phys. Rev. D **62**, 123509 2000
M. Yamaguchi, Phys. Rev. D **64**, 063503 (2001)

[5] B.A. Bassett and S. Tsujikawa, Phys. Rev. D **63**, 123503 (2001)
A. M. Green and K.A. Malik, Phys. Rev. D **64**, 021301 (2001)
F. Finelli, S. Khlebnikov, Phys. Lett. B **504**, 309 (2001); e-Print Archive: hep-ph/0009093

[6] T. Bringmann, C. Kiefer and D. Polarski, Phys. Rev. D, to appear; e-Print Archive astro-ph/0109404

[7] J.C. Niemeyer and K. Jedamzik, Phys. Rev. Lett. **80**, 5481 (1998); Phys. Rev. D **59**, 124013 (1999)

[8] D. Polarski and A.A. Starobinsky, Class. Quantum Grav. **13**, 377 (1996)
C. Kiefer, D. Polarski, and A.A. Starobinsky, Int. J. Mod. Phys. D **7**, 455 (1998)

[9] D. Polarski, Int. J. Mod. Phys. D, to appear (2001)

[10] D. Polarski and A.A. Starobinsky, Nucl. Phys. B **385**, 623 (1992)

[11] E. Bunn, M. White, Astrophys. J. **480**, 6 (1997)

[12] V. Sahni, A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000)

[13] E.V. Bugaev and K.V. Konishchev, e-Print Archive astro-ph/0103265

[14] D. Polarski, Phys. Lett. B **458**, 13 (1999)

[15] D. Polarski, Phys. Rev. D **49**, 6319 (1994)

[16] A.A. Starobinsky, JETP Lett. **55**, 489 (1992)

[17] J. Lesgourgues, D. Polarski, and A.A. Starobinsky, Mont. Not. Roy. Astron. Soc. **297**, 769 (1998)
M. Kamionkowski and A.R. Liddle, Phys. Rev. Lett. **84**, 4525 (2000)

[18] T. Bringmann, C. Kiefer and D. Polarski, in preparation